An Energy-Conserving Hair Reflectance Model

一种能量守恒的头发反射率模型

Energy-Conserving 能量守恒

摘要：

Account for 对…负有责任，对…做出解释，说明…的原因

Cone 圆锥体

contraction of specular cones 高光圆锥体的收缩

azimuthal 方位角的

Accounting for roughness in the azimuthal direction leads to an integral across the  
hair fiber which is efficiently evaluated using a Gaussian quadrature.

为了说明在方向角方向上的粗糙度，引入了一个对整根毛纤维的积分，这个积分可以高效地用一个高斯求积计算。

引言：

要想准确的计算整个头发模型的反射，那么必须先计算好单根发丝的反射。

Image synthesis 图像合成

an individual hair strand 一根单独的发丝

an analytic factored model 一个分析因子模型

grazing angles of illumination 掠射角度

This model is widely  
successful but is based on several simplifying assumptions  
that limit its accuracy for low-absorbing hair (such as light  
blond and white hair), for grazing angles of illumination, and  
for high surface roughness.

这个模型获得了广泛的成功，但它是基于几个简化的假设，这些假设限制了它对低吸收头发(如浅色金发和白发)、掠射角度和高表面粗糙度的准确性。

Nearly a decade ago approximations were chosen to yield the most efficient model possible that was capable of matching measured data.

大约十年前，人们选择了近似的方法来生成最有效的模型，以便能够匹配测量数据。

Today, we  
find it practical to revisit the application of Marschner et al.’s  
model to rough surfaces, deriving more complex scattering  
functions that have several desired properties.

今天，我们发现重温Marschner等人的模型在粗糙表面上的应用是可行的，可以推导出更复杂的散射函数，这些函数具有几个期望的性质。

2.相关工作

It has been improved upon significantly by Marschner et al. [MJC∗03] by considering variation of reflectance in the azimuthal directions and accounting for internal absorption and caustics.

Marschner等人[MJC 03]通过考虑反射在方位方向上的变化并考虑内部吸收和焦散，对其进行了显著改进。

Zinke and Weber [ZW07] introduced the formalism of Bidirectional Curve Scattering Distribution Functions (BCSDFs) and a near-field model, important for close  
rendering of hair fibers and global illumination.

Zinke和Weber [ZW07]引入了双向曲线散射分布函数(BCSDFs)的形式体系和近场模型，这对于头发纤维的近距离渲染和全局光照非常重要。

They also  
presented a far-field model which is very similar to the azimuthal component of our new model, but with a different  
treatment of azimuthal roughness.

他们还提出了一个远场模型，该模型与我们的新模型的方位分量非常相似，但对方位粗糙度的处理不同。

Ogaki et al. [OTS10] present a simulation-driven approach to generating hair reflectance functions based on  
photon tracing through translucent user-modeled hair-fiber  
micro-geometry.

Ogaki等人[OTS10]提出了一种基于模拟驱动的方法来生成头发反射率函数，该方法基于光子跟踪，通过半透明的用户建模的头发纤维微观几何形状。

Our model can be efficiently integrated within previous  
methods for computing multiple scattering within hair volumes [LV00, MM06, ZYWK08, SSD∗10]

我们的模型可以在以前的方法中有效地集成，从而计算头发体积内的多重散射[LV00, MM06, ZYWK08, SSD 10]

For an extensive review of hair shading, animation and modeling techniques  
we refer the reader to the survey by Ward et al. [WBK∗07].

要广泛回顾头发着色、动画和建模技术，我们请读者参考Ward等人的调查[WBK 07]。

Longitudinal analysis 纵向分析

Azimuthal Analysis 方位角分析

Our model incorporates a new TRRT term (shown in green).

我们的模型包含了一个新的TRRT术语(绿色表示)。

3. Factored Hair Reflectance Models 因子头发反射模型

A rich set of phenomena is predicted by the analytic model of Marschner et al. [MJC∗03] by decomposing the reflectance into separate modes of propagation.

利用Marschner等人[MJC 03]的分析模型，将反射率分解为独立的传播模式，预测出一组丰富的现象。

We review the factored pathway analysis of Marschner et al. for smooth hair fibers, which forms the basis of our new model,and subsequently investigate the method of treating surface roughness.

我们回顾了Marschner等人对光滑毛发纤维的因子通路分析，这是我们新模型的基础，随后研究了处理表面粗糙度的方法。

We adopt the notation of Marschner et al. [MJC∗03] for  
parametric hair reflectance models described as a function of incoming and reflected directions, denoted ω~ i and ω~r, respectively.

我们接受了Marschner的描述符号，对于参数化的毛发反射模型，描述为入射和反射方向的函数，分别表示为i和r。光线方向。

i(incoming)下标统一是光线入射，r(reflected)统一是光线反射。

The local coordinate system of the hair, {,,},  
has ~u tangent to the hair in the direction of growth and the  
plane spanned by {,} is called the normal plane.

毛发的局部坐标系{,,}与毛发在生长方向上有切线，由{,}所张成的平面称为法平面。

就是说是毛发的主要生长方向。

Symmetry permits a parametrization in terms of longitudinal inclinations to the normal plane , θi and θr, together with the  
relative azimuth, φ = φr - φi.

对称性允许在纵向倾斜方面对法线平面进行参数化，θi 和θr,连同相对方位角， φ = φr - φi.

θi是光线在纵向平面上的入射角，θr是光线在纵向平面上的反射角。

φi是光线在方位角平面上的入射角，φr是光线在方位角平面上的反射角。

Notation is simplified by referring to the longitudinal difference angle θd = (θr - θi)/2  
and half angle θh = (θr +θi)/2.

符号简化是指的纵向差异角度θd =(θr−θi) / 2和半角θh =(θr +θi) / 2。

The index of refraction η of  
the hair is typically fixed at 1.55.

头发的折射率η通常是固定在1.55。

Absorption inside hair is  
primarily caused by two pigments: eumelanin and pheomelanin with concentrations ρe and ρp respectively.

头发内的吸收主要由两种色素引起：真黑素和褐黑素，其浓度分别为ρe和ρp。

absorption cross-sections 截面吸收率

Given the  
absorption cross-sections for eumelanin σa,e and pheomelanin σa,p for each wavelength of light (see Section 6.1), the  
spectral absorption coefficient is µa = ρeσa,e + ρpσa,p.

给定真黑色素的截面吸收率是σa,e，褐黑素的截面吸收率是σa,p，

对于每个波长的光，光谱吸收系数为µa = ρeσa,e + ρpσa,p.

A factored reflectance model S(θi,θr,φ) exploits the Bravais properties of an idealized smooth circular hair fiber.

分解反射模型S（θi，θr，φ）利用理想光滑圆形头发纤维的Bravais特性。

The total reflectance is decomposed into a sum of contributions indexed by the number of path segments p inside the hair before exiting, p ∈ {R = 0, TT = 1, TRT = 2, TRRT = 3, ...}(see Figure 1).

总的反射被分解成了一个按光线在头发内部退出的先后顺序索引的路径片段各自的反射率的叠加。

同时光线的反射率被分解在两个平面上，一个是纵向平面Mp（反射率），一个是方向角平面Np（反射率），

For a circular fiber with a smooth surface, the first four scattering functions Sp can be solved for exactly.

对于表面光滑的圆形光纤，前四个散射函数Sp可以精确求解。

Effects from elliptical hair fibers, surface roughness, and tilted cuticle scales are introduced approximately by modifying the idealized model in an appropriate fashion.

通过对理想模型进行适当的修改，近似地引入椭圆毛纤维、表面粗糙度和倾斜角质层鳞片的影响。

4. Longitudinal Scattering 纵向散射

In this section we consider the thought experiment of observing a non-absorbing hair fiber in a uniform, white environment.

在本节中，我们考虑在均匀的白色环境中观察不吸收毛发纤维的思想实验。

The 1/η2 solid angle compressions and contractions cancel and all paths lead to white, making the hair indistinguishable from the white background.

1 /η2固定角度压缩和收缩相护抵消，所有路径都变成白色，使得头发与白色背景难以区分。

The analysis of this problem (or, specifically, an equivalent reciprocal problem) is used to show that previous extensions of Marschner et al. [MJC∗03] to rough hairs are not energy-conserving for all roughnesses and angles of illumination/viewing.

对这个问题(或者更确切地说，一个等价的倒数问题)的分析表明，Marschner等人以前对粗糙毛发的扩展(MJC 03)并不是对所有的粗糙和光照/观察角度都是能量守恒的。

We use the same analysis to demonstrate the energy conservation of our new proposed scattering function.

我们用同样的分析来证明我们新提出的散射函数的能量守恒。

4.1. The Gaussian Mp Longitudinal Scattering Function 高斯纵向散射函数Mp

Marschner et al. [MJC∗03] propose an efficient method for simulating reflectance from a hair fiber with a rough surface.

Energy-CoMarschner等人[MJC 03]提出了一种有效的方法来模拟具有粗糙表面的毛发纤维的反射率。

In the idealized case of a smooth circular hair, light  
arriving along a single incoming inclination θi produces reflected light restricted to the specular cone θr = -θi (Figure 2 left).

在理想情况下圆形光滑的头发,光到达沿着一个传入的倾向θi产生反射光局限于镜面锥θr =−θi(图2)。

This yields a longitudinal scattering function  
Mp = δ(θr + θi) for all p and is energy conserving.

这产生了一个纵向散射函数Mp =δ(θr +θi)对于所有p并且这是能量守恒的。

To simulate deviation of reflected directions out of the perfect specular cone due to roughness on the surface of the hair,Marschner et al. [MJC∗03] propose using a Gaussian function of the half-angle

为了模拟由于毛发表面粗糙导致反射方向偏离理想镜面锥，Marschner等人[MJC 03]建议使用半角的高斯函数，

Mp = g(βp;θh - αp),，where g(β;θ)

is a normalized Gaussian of longitudinal inclination θ, β is a roughness term (specifically, the standard deviation of deflection out of the specular cone in the longitudinal direction), and αp is a simple function of the tilt of the cuticle  
scales (Figure 1).

g(β;θ)是一个归一化的高斯函数，

θ是纵向的倾斜角

β是一个粗糙系数

αp是表皮鳞片倾斜的简单函数

The angular redistribution of radiance in  
Equation (3) is non-conservant for several reasons:

由于若干原因，等式（3）中的辐射度的角度重新分布是不可观的：

figure 2.

The compression and contraction of these cones involves a net change in radiance.

这些锥体的压缩和收缩涉及到光强的净变化。

For deflection into angles toward the normal plane (right) the same number of photons enter a wider cone and the radiance decreases.

对于向法线平面(右图)偏转的角度，同样数量的光子进入更宽的锥体，辐射降低

• The Gaussian in Equation (4) is normalized with respect to θ ∈ {-∞,∞} but it is evaluated with θh ∈  
{-π/2,π/2}. Also, the use of θh instead of θ doubles the  
reflected energy on average (Figure 3)

在方程(4)中的高斯函数是针对θ ∈ {-∞,∞}归一化后的结果，但是它实际是要评估θh ∈{-π/2,π/2}.使用θh替换θ会使平均放射能量翻倍。

* Deflection of light from one specular cone -θi into another θr involves a compression or contraction of the cones when θi 6= -θr (see Figure 2).

当θi-θr时，从一个镜面锥-θi到另一个θr的光的偏转涉及锥体的压缩或收缩（参见图2）。

This is only approximately accounted for by a 1/cos2 θd term in Marschneret al. [MJC∗03].

这个仅能近似的用一个的量来说明。

* Deflection near grazing illumination angles moves considerable energy into angles outside of the range θ ∈{-π/2,π/2} (angles that will never be received, thereby losing energy).

掠射照射角附近的偏转将相当大的能量移动到范围θ∈{-π/ 2，π/ 2}之外的角度（将永远不会被接收的角度，从而失去能量）。

A characterization of this non-conservant scattering can be seen by considering the total reflectance of an absorption free (µa = 0) hair fiber under unit monodirectional illumination.

characterization 特征因数

non-conservant

通过考虑单位单向照射下无吸收（μa= 0）毛发纤维的总反射率，可以看出这种非能量守恒散射的特征。

To simplify the analysis, we observe the behaviour of MR only by having R return all the energy (setting the index of refraction to ∞).

为了简化分析，我们仅通过让R返回所有能量（将折射率设置为∞）来观察MR的行为。

**4.2. An Energy-Conserving Longitudinal Scattering  
Function *Mp*** 能量守恒的纵向散射函数Mp

We derive a novel longitudinal scattering function that conservatively redistributes reflected radiance amongst directions on the sphere.

我们推导了一个新的纵向散射函数，该函数保守地重新分配了球面上不同方向的反射辐射。

This is achieved by employing sphericalGaussian convolution.

这是通过使用球面高斯卷积实现的。

For use with a factored reflectance  
model, we derive a purely-longitudinal function by averaging the behaviour in the azimuthal direction.

为了与因式反射模型一起使用，我们通过平均方位角方向的行为来推导出纯纵向函数。

We consider  
the spherical Gaussian convolution of a Dirac circle on the  
surface of a sphere.

我们考虑一个狄拉克圆在球面上的球面高斯卷积。

The longitudinal profile of the resulting distribution is our new scattering function.

得到的分布的纵向轮廓是我们新的散射函数。

We refer the  
reader to Appendix A for a complete derivation.

我们请读者参阅附录A以获得完整的推导。

Io(x) is the  
modified Bessel function of the first kind.

Io(x)是第一类修正的Bessel函数。

The shape of the  
new longitudinal scattering function is shown in Figure 4 for  
various incident angles.

新的纵向散射函数在不同入射角下的形状如图4所示。

It is asymmetric about the specular  
cone angle, exhibiting an off-specular peak similar to planar  
BRDFs, and for all roughnesses is energy conserving：

镜面椎角是不对称的，表现出类似于平面BRDF的非镜面峰值，并且对于所有的粗糙度都是能量守恒的。

Note that all incidence and exitance factors are contained in  
the one reciprocal function (Equation 7).

请注意，所有入射和退出因子都包含在一个倒数函数中（公式7）。

This is convenient  
in a rasterization framework, the contribution of a given fiber  
to a given pixel being proportional to its coverage C of the  
pixel and to the final scattering function S (Equation 1) ,C S(θi,θr,φ)

这在光栅化框架中是方便的，给定光纤对给定像素的贡献与其像素的覆盖范围C和最终散射函数S（等式1）成比例，C S(θi,θr,φ)。

The unprojected length of the hair within a  
pixel, the hair width, the cos θi and 1  
cos2 θd factors required of  
previous models are absent. The transformation to a BCSDF  
is straightforward.

头发在一个像素内的未投影长度，宽度，先前模型所需的cosθi和1/cos2θd因子都是缺少的。 转变为BCSDF非常简单。

5. Azimuthal Scattering 方位角散射

Marschner et al. [MJC∗03] analyze the azimuthal scattering  
behaviour inside a fiber independently from the longitudinal  
behaviour by exploiting Bravais properties of smooth cylinders.

Marschner等人[MJC 03]利用光滑圆柱的Bravais特性，独立于纵向特性分析纤维内部的方位散射行为。

Using a modified index of refraction η0 = √ηcos 2-sin θd2 θd ,  
the general analysis can be restricted to an equivalent analysis in the normal plane only.

使用修正折射率η0 = √ηcos 2-sin θd2 θd , ，一般分析可以仅限于正常平面中的等效分析。

Marschner et al. [MJC∗03] use a cubic approximation of  
Equation (8) when solving for h, reducing the complexity  
of higher-order terms and offering an approximate method  
of solution for any desired reflection mode p.

Marschner等人[MJC 03]使用的是立方体近似

式(8)在求解h时，降低了高阶项的复杂度，为任意期望的反射模态p提供了近似的求解方法。

However, the  
accuracy of this approximation has not been validated for  
large p or for low relative indices of refraction (which arises  
for hair under water).

然而，对于较大的p值或较低的相对折射率(水下毛发的相对折射率)，这种近似的准确性尚未得到验证。

In addition to the complexity of requiring root solvers, the  
approach of solving for h is further limited by singularities  
caused by caustics, requiring ad-hoc user-defined parameters to control them.

除了需要根求解器的复杂性之外，求解h的方法还受到焦散引起的奇点的限制，需要特定的用户定义参数来控制它们。

Additionally, the consideration of discrete azimuthal paths independent of roughness cannot account for the spreading of exitant directions and color shifts  
that occur as roughness increases.

另外，考虑与粗糙度无关的离散方位角路径不能解释随着粗糙度增加而出现的出射方向和色移的扩展。

We present an alternative  
azimuthal scattering function that extends easily to higher  
order scattering terms, includes caustics in a consistent stable fashion, and exhibits roughness-dependent color shifts.

我们提出了一个可选的方位散射函数，它可以很容易地扩展到高阶散射项，以一致稳定的方式包含焦散，并显示出与粗糙度相关的颜色偏移。

5.1. Roughened Azimuthal Scattering Functions Np 带有粗糙度的方位散射函数Np

We simulate the effects of surface roughness by assuming  
a Gaussian distribution of deflections in the normal-plane.

我们通过假设正常平面中的偏转的高斯分布来模拟表面粗糙度的影响。

Each offset h on the fiber produces a continuous distribution of exitant azimuths Dp(φ-Φ(p,h)).

每一个在纤维上的偏移量h产生了一个出射方位Dp(φ-Φ(p,h))的连续分布。

These distributions  
are Gaussian in φ and centered about the discrete azimuths  
Φ(p,h) predicted by a smooth hair (Figure 7).

这些分布在φ中是高斯分布，并且以光滑毛发预测的离散方位角Φ（p，h）为中心（图7）。

The total exitance is found by integrating over the entire fiber  
Np  
(φ) = 1  
2 Z-11 A(p,h)Dp(φ - Φ(p,h))dh (10)  
where A(p,h) is the attenuation along each path due to absorption and Fresnel [MJC∗03].

总的出射量可以通过对整个纤维进行积分得到，

Np(φ) = 1/2 Z-11 A(p,h)Dp(φ - Φ(p,h))dh (10),

A(p,h)是一个各条路径的衰减系数，是由于吸收和菲尼尔效应。

We employ a specialized  
normalized Gaussian function D  
p with an equivalence of all  
multiples of 2π.

我们使用了一个特效的归一化的高斯函数Dp，对于所有2π的倍数都是等价的。

Detection at all multiples of 2π is important in order to account for all exitant light, which may undergo several complete azimuthal revolutions inside the hair for p > 1.

在所有2π的倍数下检测是重要的，以便考虑所有的出射光，其可以在头发内经历几个完全的方位角旋转，对于p> 1。

The roughness in the normal plane can use the same standard deviations βp used with the longitudinal functions or  
use separate parameters for producing anisotropic roughness.

法向平面的粗糙度可以使用相同的标准差βp纵向函数或使用单独的参数用于产生各向异性的粗糙度。

As the roughness β approaches 0 the Gaussian detector approaches a Dirac delta function and the scattering  
functions of Marschner et al. [MJC∗03] are recovered starting with Equation (10).

随着粗糙度β趋近于0，高斯检测器接近狄拉克函数并且Marschner等人的散射函数也从等式[10]开始恢复。

In addition to spreading light wider,  
roughness-dependent color shifts are exhibited when using  
our new azimuthal scattering function (Figure 8).

除了将光传播得更广外，在使用我们的新方位散射函数时，还显示出与粗糙度相关的颜色偏移(图8)。

6. Final Model and Implementation 最终的模型和实现

Figure 8:

Azimuthal transmittance color 方位透光率的颜色

Including roughness in  
the azimuthal direction spreads energy wider.

把方位方向的粗糙度包括在内，能量传播范围更广。

The continuum of absorption lengths contributing to each exitant direction φ causes a roughness-dependent color shift.

对每个光线射出方向φ有贡献的吸收长度的连续长度引起与粗糙度有关的色移。

For completeness, we provide the remainder of the model  
(which differs from Marschner et al. [MJC∗03] in some respects).

为了完整起见，我们提供了模型的其余部分

(这在某些方面与Marschner等人[MJC 03]不同)。

The attenuation term for R is a special case

R的衰减项是一个特例

where F(η,θ) is Fresnel reflectance and is evaluated using  
the half-angle for physical consistency.

F(η,θ)是菲尼尔反射，为了物理的一致性用半角来求值。

We note that the Bravais Fresnel terms proposed by Marschner et al. [MJC∗03] are unnecessary—all other terms can use

我们注意到，由Marschner等人提出的Bravais Fresnel术语是不必要的——所有其他术语都可以使用

which share a common classical Fresnel evaluation

具有相同的经典菲涅耳求值

The absorption term 吸收项

uses the reduced absorption coefficient 使用降低的吸收系数

Effects of cuticle scale tilting can be added by evaluating the longitudinal scattering functions with Mp(vp,θi,θr - αp).

通过用Mp（vp，θi，θr - αp）评估纵向散射函数，可以增加角质层倾斜的效果

Note that this is only approximate—the change in length of successive internal bounces due to tilted scales (shown in Figure 1a) is not accounted for.

请注意，这只是近似值 - 由于倾斜的刻度（如图1a所示）导致的连续内部反弹长度的变化未被考虑。

The Gaussian detector(Equation 11) has a closed form representation in terms of the third Eliptic Theta function.

in terms of 依据，按照

依据第三个椭圆雅各比函数，高斯检测器（等式11）具有一个闭合形式表示。

In practice it is easiest to implement it with a finite range of k, since higher order terms are negligible for reasonable roughness levels (β < 80◦).

在实践中，使用有限范围的k实现它是最容易的，因为对于合理的粗糙度水平（β<80°），高阶项可忽略不计。

We evaluate Equation (10) using a Gaussian quadrature.

我们用高斯求积法求方程(10)的值。

We found a quadrature of order 35 is sufficient for all but  
very smooth hairs (β < 2◦).

我们发现35阶的正交对于除了非常光滑的毛发（β<2°）都足够了。

Furthermore, symmetry can be used to reduce cost by integrating over h ∈ {0,1} and extending the  
Gaussian detector to detect at both φ and -φ.

此外，通过积分超过h∈{0,1}并扩展高斯检测器以在φ和-φ处检测，可以使用对称性来降低成本。

Non-Gaussian roughness can be simulated by approximating a general deflection distribution function as a Gaussian sum.

可以通过将一般偏转分布函数近似为高斯和来模拟非高斯粗糙度。

6.1. RGB Melanin Absorption Cross-Sections RGB黑色素吸收横切面

Starting with the analytic approximations for melanin absorption cross-sections given by Donner and Jensen [DJ06],we computed absortion colors using 40 spectral bands,mapping to sRGB based on a D65 illuminant.

从Donner和Jensen [DJ06]给出的黑色素吸收截面的分析近似开始，我们使用40个光谱带计算吸收颜色，基于D65光源映射到sRGB。

We found that RGB cross-sections could closely, but not perfectly,match the colors predicted by the spectral model.

我们发现RGB横截面可能与光谱模型预测的颜色非常接近，但并不完美。

The values we found are σa,e = {0.419,0.697,1.37} and σa,p = {0.187,0.4,1.05} (with units proportional to R2 such that concentrations near 1/R3 give black hair; R being the radius of the hair).

（单位与R2成正比，使浓度接近1/R3时，头发呈黑色; R为半径 头发）

The horizontal bars at the top and bottom of the figure  
compare spectral absorption calculations to the approximate RGB equivalents.

图的顶部和底部的水平条将光谱吸收计算与近似RGB当量进行比较。

The match is quite close, making the horizontal edge between the two difficult to notice.

匹配非常接近，使得两者之间的水平边缘很难被注意到。

Figure 9:

Transmitted color through eumelanin and  
pheomelanin mixtures as a function of thickness (increasing to the right).

通过真黑素和非真黑素混合物投射颜色作为浓度度的函数(向右增加)。

Strips at the top and bottom show pure eumelanin and pheomelanin, respectively, with the top half of  
each strip showing the spectral reference, and the bottom  
half showing the RGB approximations.

顶部和底部的条带分别显示纯的真黑素和褐黑素，每个条带的上半部分显示光谱参考，下半部分显示RGB近似。

7. Results 结果

Figure 10:

A range of results using our model with varying mixtures of eumelanin and pheomelanin.

用我们的模型对不同的真黑素和非真黑素混合物进行了一系列的研究。

The top row shows pure eumelanin, the bottom row shows pure pheomelanin, and total concentration of both increases to the right.

最上面一行显示的是纯的真黑素，最下面一行显示的是纯的黑色素，这两种物质的总浓度都往右边增加。

Figure 12:

Various natural hair colors using our new reflectance model with Dual Scattering. Glints are exhibited in an energyconserving fashion with no extra control parameters.

使用我们新的反射模型与双散射呈现的多种天然发色。在没有额外控制参数的情况下，闪烁以一种能量守恒的方式表现出来。

In this section we present images obtained using a Pixar Photorealistic Renderman implementation of our model. We compute multiple scattering using the Dual Scattering method [ZYWK08] into which our model integrates seamlessly.

我们使用双散射方法[ZYWK08]计算多次散射，我们的模型无缝集成。

Figure 11 compares a Dual Scattering simulation of light hair using both Gaussian longitudinal functions (MR = g(βR;2θh) to avoid energy doubling) and our  
spherical Gaussian convolution longitudinal function.

图11比较了使用高斯纵向函数（MR = g（βR;2θh）以避免能量倍增）和我们的球面高斯卷积纵向函数的光发的双散射模拟。

Light spreads farther into darker regions with our new scattering function because Dual Scattering approximates multiplescattering events as a single computation with a large effective roughness.

利用我们的新散射函数，光线进一步扩散到更暗的区域，因为双散射将多个散射事件近似为一个具有较大的有效粗糙度的单个计算。

Such high roughnesses are where the Gaussian longitudinal function loses signifant energy (for most  
angles, as seen in Figure 3).

如此高的粗糙度是高斯纵向函数失去大量能量的地方（对于大多数角度，如图3所示）。

We demonstrate a full range of hair colors derived from spectrally-matched melanin concentrations, shown in Figure 10. Figure 12 shows a further example of natural, physically-based color, also demonstrating energy conservant glints that do not require any smoothing controls.

我们展示了来自光谱匹配的黑色素浓度的全系列染发颜色，如图10所示。图12显示了自然的，基于物理的颜色的另一个例子，也展示了不需要任何平滑控制的能量保护闪光。

Glints can easily be adjusted by modifying the physical  
properties of the hair fibers, such as rotation and eccentricity similar to Marschner [MJC∗03]

通过改变头发纤维的物理特性，例如旋转和偏心，可以很容易地调整毛刺，类似于Marschner [MJC \* 03]

8. Conclusions and Future Work 结论和未来的工作

An area for future work is phenomenological comparisons with this new model and its easy extension to non-uniform chromophore distributions, such as including a non-absorbing medulla.

未来工作的一个领域是将现象学与这个新模型进行比较，并将其易于推广到非均匀色素体分布，例如包括一个不吸收的髓质。

Preliminary experiments with this idea led to subtle shifts in color,  
requiring measurements to validate its practicality.

这个想法的初步实验导致了颜色的微妙变化，需要测量来验证其实用性。

The energy analysis of TRRT and higher order terms suggests that  
these are not the cause for diffusive colored reflectance observed in hair, as previously suggested.

TRRT和高阶项的能量分析表明，这些并不是像之前所指出的那样，在头发中观察到的扩散彩色反射率的原因。

We believe this phenomena must arise due to inner-core scattering, which also  
desaturates the primary terms.

我们认为，这一现象的产生一定是由于内核散射，这也使主要项的饱和度降低。

A possible extension to feathers seems promising (Figure 13).

对羽毛的一个可能性扩展似乎很有希望（图13）。

9. Acknowledgments